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EFFECT OF A LOCAL INCREASE IN CONDUCTIVITY OF THE E-LAYER OF THE IONOSPHERE ON THE S_q- VARIATION OF THE EARTH'S MAGNETIC FIELD *

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ABSTRACT.

Maxwell equations for a stationary current are utilized for the computation of the effect on S_q-currents and S_q-variation of the geomagnetic field of local increase in conductivity of the E-layer of the ionosphere. Expressions for a current function within and without the region of local conductivity increase are obtained. Conducted also is an additional such computation for a system that would appear for a circular region of 1000 km radius, for an increase by 100% of the conductivity at the center of the region, and for conductivity decrease toward the region's boundary to the unperturbed value according to the parabolic law and density of unperturbed S_q-currents equal to $\sim 1.5 \cdot 10^{-4}$ un. CGSM per 1 cm meridian length.

It is obtained that the S_q-variation of the X-component increases at points lying under the region of increased conductivity, and decreases at points that are not lying under that region. The variation of S_q in the Z-component is increased by the southern current eddy and decreased by the northern. The intensity of the complementary current through the region would equal $\sim$ 7000 a, which constitutes $\sim$ 30% of the strength of unperturbed S_q-currents through that region. The value of the X-component at the point under

* Vliyaniye lokal'nogo povysheniya provodimosti E-sloya ionosfery na S_q - variatsiyu magnitnogo polya Zemli.

under the center of the region is equal to -3.5γ.

The values of the X-component of the complementary current's magnetic field are computed along the meridian passing under the center of the region, and the values of the Y-component along the parallel passing under the seat of the current eddy. As the conductivity increases in the center by 300%, the amplitudes of components' variations along these cross-sections are respectively equal to 10.5γ and 6γ.

* * *

COVER-TO-COVER TRANSLATION

It was assumed in references [1 — 6] that one of the causes of geomagnetic effects of nuclear explosions in the lower atmosphere was the increase of S_q-variations of the Earth's magnetic field above the explosion regions on account of ionization density increase in the portions of the E-layer above the explosions' points, the latter leading to local increase of conductance in these portions. It was assumed in reference [7] that the subsequent decrease of H- and Z-components in the magnetic effect of the Tungusska meteorite was caused by the increase of S_q-variation in Irkutsk as a consequence of local increase in conductivity of the E-layer of the ionosphere above the territory of Eastern Siberia. The conductivity increase was caused by thermal ionization of the air by the shock wave at the ionosphere E-layer's level.

To verify these assumptions, it appears to be useful to solve the problem of computing the variations of the Earth's magnetic field, which would be likely to occur at local conductivity increase in the E-layer. The reverse problem — that of the geomagnetic effect of local decrease in conductivity of the E-layer — was resolved in references [8 — 9]. These results were used for the study of the geomagnetic effects of solar eclipses. The methods

for resolving the reverse problem may be utilized for the solution of the current problem of interest to us. Thus, it was proposed in [3] to apply the method of reference [9].

We carried out computations of a current system that would occur as a result of local conductivity increase in the E-layer of the the ionosphere, and we have computed the X- and Y- components of the magnetic field intensity of that current system at the Earth's surface for a single particular case. To do that we used the method for solving the reverse problem applied in reference [9], modified in that the original equation is resolved for the current function, as this was done for example in [10] and not for the electric field's potential.

Let us postulate that a local conductivity increase takes place in a circular portion of the E-layer of the ionosphere, whose radius would be a. The conductivity increases toward the center of the circular region and decreases toward its boundaries according to the parabolic law:

$$K = K_0\left(1 + \gamma\frac{a^2 - r^2}{a^2}\right), \qquad (1)$$

where K is the integral effective conductivity of the E-layer; at the boundary and beyond the region the conductivity is constant and equal to K_0, r is the radial distance from the center of the circle, γ is a positive constant. The initial equations

$$\mathrm{Rot}\,\mathbf{E} = 0, \quad \mathrm{Div}\,\mathbf{j} = 0, \quad \mathbf{j} = K\mathbf{E}, \qquad (2)$$

where E and j respectively are the vectors of electric field strength and of current density. The equation Div j = 0 allowed to introduce a current function Ψ, linked with the current density by the correlation

$$\mathbf{j} = -[\bar{n}\,\mathrm{Grad}\,\Psi], \qquad (3)$$

where $\bar{n}$ is the unitary vector of the normal to the current layer

(Rot, Div, Grad are the respective designations for surface rotor, divergence and gradient). Combining formulae (2) and (3), we obtain

$$\mathrm{Div}\left(\frac{1}{K}\,\mathrm{Grad}\,\Psi\right)=0, \tag{4}$$

or in the polar system of coordinates, whose origin is situated at the center of the circle :

$$\frac{1}{K}\left(\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}\right)+\frac{\partial}{\partial r}\left(\frac{1}{K}\right)\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{K}\right)\frac{\partial\Psi}{\partial\theta}=0. \tag{5}$$

The angle θ is counted from the direction of the unperturbed S_q - current. Solution of (5) is sought in the form

$$\Psi_1=\psi_1(r)\sin\theta \quad r<a, \quad \Psi_2=\psi_2(r)\sin\theta \quad r>a. \tag{6}$$

Substituting (1) and (6) into (5), we obtain that $\psi_1(r)$ and $\psi_2(r)$ satisfy the following differential equations :

$$\begin{aligned}&\frac{d^2\psi_1}{dr^2}+\frac{1}{r}\left(\frac{1+cr^2}{1-cr^2}\right)\frac{d\psi_1}{dr}-\frac{\psi_1}{r^2}=0 \quad r<a,\\ &\frac{d^2\psi_2}{dr^2}+\frac{1}{r}\frac{d\psi_2}{dr}-\frac{\psi_2}{r^2}=0 \quad r>a,\end{aligned} \tag{7}$$

where $c=[\gamma/(1+\gamma)]\frac{1}{a^2}$.

The solution of equations (7) is found at conditions

$$\psi_1=\psi_2, \quad \frac{d\psi_1}{dr}=\frac{d\psi_2}{dr} \text{ at } r=a$$
$$I_1 \text{ is finite at } r=0, \; I_2=I_0 r \text{ at } r\to\infty \tag{8}$$

where I_0 is the density of the unperturbed S_q- currents.

The solution of the first of equations (7), satisfying the condition of finiteness for $r=0$, is sought according to [11] in the form

$$\Psi_1=r^k\sum_{s=0}^{\infty}\alpha_s r^s, \tag{9}$$

where k and α_s are constants. Preliminarily the coefficient at

the first derivative in the equation is represented in the form of the series:

$$\frac{d^2\psi_1}{dr^2} + \frac{1}{r}(1 + 2cr^2 + \cdots + 2c^{n/2}r^n + \cdots)\frac{d\psi_1}{dr} - \frac{\psi_1}{r^2} = 0. \qquad (10)$$

Having determined the first and the second derivatives and having substituted them into (10), we shall obtain for $\alpha_0 \neq 0$, $k = 1$

$$\alpha_1 = \alpha_3 = \ldots = \alpha_s = \ldots = 0$$

$$\alpha_s = c^{s/2}\prod_s \frac{s^2 - 4s + 2}{s(s+2)}\alpha_0, \qquad (11)$$

where s — is an odd number, beginning with 2, α_0 is a constant derivative. The solution of the equation after computation of the first five coefficients α_s has the form

$$\Psi_1 = \alpha_0 r\left\{1 - \frac{1}{4}\frac{\gamma}{1+\gamma}\frac{r^2}{a^2} - \frac{1}{48}\left(\frac{\gamma}{1+\gamma}\right)^2\frac{r^4}{a^4} - \frac{7}{1152}\left(\frac{\gamma}{1+\gamma}\right)^3\frac{r^6}{a^6} - \right.$$
$$\left. - \frac{217}{6912}\left(\frac{\gamma}{1+\gamma}\right)^4\frac{r^8}{a^8} + \cdots + \left(\frac{\gamma}{1+\gamma}\right)^{s/2}\frac{r^s}{a^s}\prod_s\frac{s^2-4s+2}{s(s+2)} + \cdots \right. \qquad (12)$$

The series in parentheses is converging for all values of r inside the region of radius a. Indeed, inasmuch as $r < a$ and

$$\prod\frac{s^2 - 4s + 2}{s(s+2)} < 1,$$

every term of this series is smaller than the corresponding term of the series

$$1 - \frac{\gamma}{1+\gamma} - \cdots - \left(\frac{\gamma}{1+\gamma}\right)^{s/2} - \cdots, \qquad (13)$$

which is a geometrical progression with a denominator $[\gamma/(1+\gamma)] < 1$, and this, according to [11] is the criterion of series' (12) convergence.

The second equation (7) is reduced upon substitution of $(d\psi_2/dr) = \psi_2 z$ [12] to a differential equation of the Rikatti type

$$r^2\left(\frac{dz}{dr} + z^2\right) + rz - 1 = 0, \qquad (14)$$

which, by substitution $z = (1/r + 1/y)$ is brought in its turn to the linear differential equation of the first order

$$\frac{dy}{dr} - \frac{3}{r} y = 1. \tag{15}$$

According to [12] the general integral of this equation is sought for by the formula

$$y = \exp\left\{\int \frac{3}{r}\, dr\right\}\left[\int \exp\left(-\int \frac{3}{r}\, dr\right) dr + C_0\right], \tag{16}$$

where C_0 is an arbitrary constant.

After returning to the variable ψ_2 we obtain the solution of the equation in the form:

$$\psi_2 = \frac{c_1}{r} - c_2 r. \tag{17}$$

Having determined the constants α_0, c_1 and c_2 from (8), we have

$$\begin{aligned} \psi_1 &= -2I_0 \frac{1+F_1}{2+2F+G} r \quad r < a, \\ \psi_2 &= -I_0\left[r - \frac{Ga^2}{(2+2F+G)r}\right] \quad r > a, \end{aligned} \tag{18}$$

$$F_1 = \sum_{s=2}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^{s/2} \left(\frac{r}{a}\right)^s \prod_s \frac{s^2-4s+2}{s(s+2)},$$

$$F = \sum_{s=2}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^{s/2} \prod_s \frac{s^2-4s+2}{s(s+2)},$$

$$G = \sum_{s=2}^{\infty} s \left(\frac{\gamma}{1+\gamma}\right)^{s/2} \prod_s \frac{s^2-4s+2}{s(s+2)}. \tag{19}$$

In (19) the summation is carried out only by even values of s. On the basis of (6), we have

$$\begin{aligned} \Psi_1 &= -2I_0 \frac{1+F_1}{2+2F+G} r \sin\theta \quad r < a, \\ \Psi_2 &= -I_0\left[r - \frac{Ga^2}{(2+2F+G)r}\right] \sin\theta \quad r > a. \end{aligned} \tag{20}$$

The current function of the complementary current system appearing as a result of local increase in the conductivity of the E-layer of the ionosphere, was found by substraction of the current function of unperturbed S_q-current, taken as equal to $\Psi_0 = -I_0 r \sin\theta$ from (20):

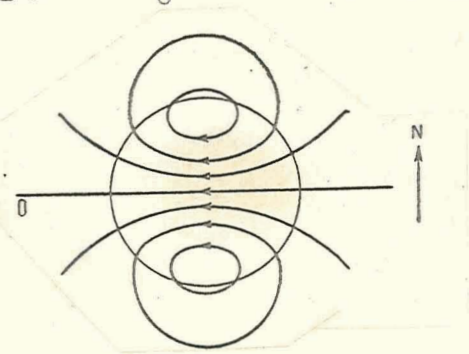

Fig. 1.

$$\Psi_1 - \Psi_0 = I_0 \frac{2F + G - 2F_1}{2 + 2F + G} r \sin\theta \qquad r < a, \tag{21}$$

$$\Psi_2 - \Psi_0 = I_0 \frac{G}{(2 + 2F + G)} \frac{a^2}{r} \sin\theta \qquad r > a.$$

Formulae (21) allow to compute the complementray current systems for the concrete values of the parameters a, γ, I_0. Presented is in Fig. 1 the form of the current system, which would appear for a circular region's radius of 1000 km, for a conductivity increase at the center of the circle by 100% greater in comparison with its normal value for the E-layer ($\gamma = 1$) and for a density of the unperturbed S_q- current equal to $\sim 1.5 \cdot 10^{-5}$ un.CGSM. The center of the circular region lies at about 60° N geographical latitude and on the meridian corresponding to 11 00 hours local solar time. It follows from Fig. 1 that at points not lying under the region of increased conductivity, the complementary current system weakens the S_q-variation in the X-component (has a horizontal component directed sidewise in a direction opposite to the unperturbed S_q-variation in the X-component). The S_q-variation in the X-component is strenghtened by the southern current eddy and is weakened at points situated under the northern current eddy. The S_q-variation in the D-component is stengthened under the western half of the low-latitude eddy and under the eastern half of the higher-latitude eddy of the complementary current system.

A complementary current, equal to ~ 7000 a, flows through the circular region between the focal points, which represents $\sim 30\%$ of the strength of the unperturbed S_q-current flowing through the same zone.

The calculation of the components of the intensity of the complementary current's magnetic field will be carried out by means of the formulae :

$$X \approx -\frac{\partial W_0}{\partial x}, \quad Y \approx -\frac{\partial W_0}{\partial y}, \qquad (22)$$
$$W_0 = 2\pi\Psi_{1,2},$$

where W_0 is that current's magnetic field potential.

Such current system would create on the surface of the Earth a magnetic field variation of the order of several gammas. Thus, on the basis of (21) — (22) the value of the field's X-component at the point situated under the center of that region, is

$$X_0 = -0{,}37 \cdot 2\pi I_0 \approx -3{,}5\gamma. \qquad (23)$$

Presented are in Fig. 2 the values of that current's X-component for the points, disposed along the geographical meridian that passes under the center of the region of increased conductivity. The computation of these values were effected by formulae (21) — (22) for the same values of parameters a and I_0 and coordinates of the center of the region, but the latter — at $\gamma = 3$. The values of the Y-component of that current's magnetic field along the parallel passing under the seat of the southern current eddy are also plotted in Fig. 2. The zero points on these graphs are respectively related to the point under the center of the region of local conductivity increase and under the seat of the southern current eddy. The X-component varies from 7γ at the point

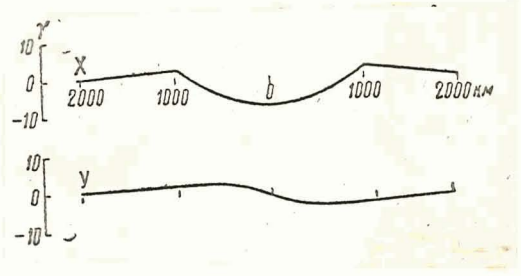

Fig 2.

under the center of the region to $+3\gamma$ at the boundary of the region and then decreases to 1γ at the distance of 2000 km from the center of the region. The Y-component is respectively equal to $+3\gamma$ and -3γ at the boundary of the region to the west and to the east of the seat of the southern eddy current.

In conclusion let us make some remarks on the results of solution of the reverse problem in [9]. The authors of that work solved the problem of computing the geomagnetic effect of local conductivity decrease in the E-layer, applying these results to the explanation of the effect upon the geomagnetic field of solar eclipses.

Formulae for the electric field potential are brought out at page 128 ref [9]:

$$\begin{aligned} \Phi_1 &= -\frac{I_0}{K_0}\frac{2}{(2F+G)}F_1 r\cos\theta && r<a, \\ \Phi_2 &= -\frac{I_0}{K_0}\left(1-\frac{G}{2F+G}\frac{a^2}{r^2}\right)r\cos\theta && r>a. \end{aligned} \tag{24}$$

Meanwhile, it is rather easy to derive from the above-expounded, that formulae for the potential have the form

$$\begin{aligned} \Phi_1 &= -\frac{I_0}{K_0}\frac{2(1+F_1)}{2+2F+G}r\cos\theta && r<a, \\ \Phi_2 &= -\frac{I_0}{K_0}\left(1-\frac{G}{2+2F+G}\frac{a^2}{r^2}\right)r\cos\theta && r>a. \end{aligned} \tag{25}$$

and differ from formulae (24) by the sum in the denominator. The utilization of formulae (24) in place of more precise formulae (25) leads to overrating the current function and the values of magnetic field intensity by nearly 100%. The utilization of formulae (25) would lead to a more significant departure between the computed and observed values of magnetic field intensity than what follows from the final results of reference [9]. Besides, I_o was taken

equal to $\sim 10^{-4}$ un. CGSM during the computations in [9], which corresponds to a current of 10^4a, flowing through a region 1000 km wide. This is 5 times greater than the usually admitted value of S_q- currents' density for the moment of time and coordinates of the region of decreased consductivity considered in [9], The normal value is $\sim 2 \cdot 10^{-5}$ un. CGSM [13]. This circumstance also led to overrating the current function and the intensity values in [9].

In conclusion I express my gratitude to Yu. D. Kalinin for his counsel and remarks put to use during the conducting of the present work.

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*** THE END ***

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